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(p, q) 5-Branes in non-zero B-field

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Abstract: We consider type-IIB (p, q) 5-branes in constant non-zero background tensor potentials, or equivalently, with finite constant field strength on the brane. At linear level, zero-modes are introduced and the physical degrees of freedom are found to be parametrised by a real 2- or 4-form field strength on the brane. An exact, $SL(2, \mathbb{Z})$-covariant solution to the full non-linear supergravity equations is then constructed. The resulting metric space-times are analysed, with special emphasis on the limiting cases with maximal values of the tensor. The analysis provides an answer to how the various background tensor fields are related to Born-Infeld degrees of freedom and to non-commutativity parameters.

Keywords: Nonperturbative Effects, D-branes, String Duality, Gauge Symmetry.

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1. Introduction

In investigating the structure of M/string theory, various extended non-perturbative objects, "branes", have proved to be very important [7]. These objects arise as solitonic solutions to the low-energy effective theories [8, 11], i.e. supergravities in eleven or ten dimensions. The physical degrees of freedom of the branes arise as zero-modes around the solitonic solutions [6, 1] in analogy with the ordinary monopole case [4] (see however ref. [1] for some important differences due to whether the theory contains gravity or not). In a previous paper [1], we generalised the Goldstone prescription for generating zero-modes to the case of tensor fields. This enabled us to treat all zero-modes on an equal footing and also showed us exactly where the zero-modes sit in the target space fields. In ref. [2] we continued to explore the zero-modes beyond linear level and were able to find exact solutions to the full non-linear supergravity equations for the D3 and M5 branes with finite constant field strength, or, equivalently, in the background of a constant tensor potential. This paper is the natural continuation of this work as we now turn to the more difficult case of type-IIB (p, q) 5-branes.

In contrast to the D3 and M5 brane cases [2], where we a priori had strong reasons to expect analytic solutions, we now have the additional complication of a real 4-form field strength on the brane, which could make things worse [3]. In addition, the object used to parametrise the zero-modes, a real 2- or 4-form, does not have any duality property as in the previous D3 and M5 brane cases. This means that the ansatz for the finite solution will contain more terms and that the algebra will be much more involved.

In section 2, we will review some properties of type-IIB supergravity that we will use and also write down the NS 5-brane solution, which will be our starting point. The zero-mode solution, which gives the parametrisation of the zero-modes, is obtained in section 3. In section 4, we solve the full non-linear supergravity equations and in section 5 we analyse the resulting metric space-times. We end by discussing the obtained solution and some possible implications.

2. Preliminaries

The type-IIB supergravity in ten dimensions has an $\mathrm{SL}(2,\mathbb{R})$ invariance (which is broken to $\mathrm{SL}(2,\mathbb{Z})$ by quantum effects) and contains two scalars in the coset space $\mathrm{SL}(2,\mathbb{R})/\,\mathrm{U}(1)$ (the dilaton ϕ and the axion χ), a selfdual 5-form field strength $H_{(5)}$, which is an $\mathrm{SL}(2,\mathbb{R})$ singlet, and a real $\mathrm{SL}(2,\mathbb{R})$ doublet of 3-form field strengths $H_{(3)r} = dC_{(2)r}$ $(r = 1,2)$, corresponding to the NS-NS and R-R field strengths respectively. However, there exists a formulation, which makes the $\mathrm{SL}(2,\mathbb{R})$ covariance manifest [9]. We will use the notation of refs. [13, 14]. The scalars are described by a complex doublet, $\mathcal{U}^r$, which obey the following $\mathrm{SL}(2,\mathbb{R})$-invariant constraint

$$\frac{i}{2}\epsilon_{rs}\mathcal{U}^r\bar{\mathcal{U}}^s = 1 \tag{2.1}$$

(we use conventions where $\epsilon^{12} = 1$, $\epsilon_{12} = -1$). Gauging the $\mathrm{U}(1)$ leaves two physical scalars which can be obtained through the projective invariant $\tau = \mathcal{U}^1/\mathcal{U}^2 = \chi + i\,e^{-\phi}$. One advantage of the covariant formulation is that the doublet of scalars transforms in a simple way under $\mathrm{SL}(2,\mathbb{R})$, whereas the physical scalars transform in a more complicated way via the projective invariant τ. The left-invariant $\mathrm{SL}(2,\mathbb{R})$ Maurer-Cartan 1-forms are

$$Q = \frac{1}{2}\epsilon_{rs}d\mathcal{U}^r\bar{\mathcal{U}}^s\,, \qquad P = \frac{1}{2}\epsilon_{rs}d\mathcal{U}^r\mathcal{U}^s\,. \tag{2.2}$$

Under the local $\mathrm{U}(1)$ transformation $\mathcal{U}^r \to \mathcal{U}^r e^{i\theta}$ (where the $\mathrm{U}(1)$ charge is normalised to 1 for the scalar doublet), the 1-forms transform as $Q \to Q + d\theta\,,\ P \to P\,e^{2i\theta}$, i.e. Q is a $\mathrm{U}(1)$ gauge field and P has $\mathrm{U}(1)$ charge 2. The Maurer-Cartan equations are

$$DP = 0\,, \qquad dQ - iP\wedge\bar{P} = 0\,, \tag{2.3}$$

where the covariant derivative $D = d - ieQ$ acts from the right (although fermions are not treated in this paper, we stick to standard superspace conventions), and e is the $\mathrm{U}(1)$ charge. The contravariant doublet of scalars can be combined with a

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covariant doublet to yield an $\mathrm{SL}(2,\mathbb{R})$-invariant object. The doublet can of course be retrieved from the scalar doublet and the $\mathrm{SL}(2,\mathbb{R})$-invariant object, e.g.,

$$\mathcal{H}_{(3)} \equiv \mathcal{U}^r H_{(3)r}\,, \qquad H_{(3)r} = \epsilon_{rs}\,\mathrm{Im}(\mathcal{U}^s \bar{\mathcal{H}}_{(3)})\,. \tag{2.4}$$

Using this formalism, the equations of motion can be written as

$$\begin{aligned} D{*}P + i\,\mathcal{H}_{(3)} \wedge *\mathcal{H}_{(3)} &= 0\,,\\ D{*}\mathcal{H}_{(3)} - i * \bar{\mathcal{H}}_{(3)} \wedge P - 4\,i\,H_{(5)} \wedge \mathcal{H}_{(3)} &= 0\,, \end{aligned} \tag{2.5}$$

and the Bianchi identities are

$$\begin{aligned} D\mathcal{H}_{(3)} + i\,\bar{\mathcal{H}}_{(3)} \wedge P &= 0\,,\\ dH_{(5)} - \frac{i}{2}\mathcal{H}_{(3)} \wedge \bar{\mathcal{H}}_{(3)} &= 0\,. \end{aligned} \tag{2.6}$$

We also have the Einstein equations

$$R_{MN} = 2\bar{P}_{(M}P_{N)} + \bar{\mathcal{H}}_{(M}{}^{RS}\mathcal{H}_{N)RS} - \frac{1}{12}g_{MN}\bar{\mathcal{H}}_{RST}\mathcal{H}^{RST} + \frac{1}{6}H_{(M}{}^{RSTU}H_{N)RSTU}\,. \tag{2.7}$$

In type-IIB we have an NS 5-brane, which couples magnetically to $C_{(2)1}$ and a D5-brane, which couples magnetically to $C_{(2)2}$ (these are of course not the only tensor couplings — any (p,q) 5-brane couples to all tensors in type-IIB supergravity, as can be seen from the known D-brane actions [5]). Here we conventionally denote the charge doublet (p,q), in the covariant formalism this becomes p_r $(r = 1,2)$, and for a classical 5-brane these are just real numbers. Due to the existence of (p,q) strings [10], the charges will be quantised and for the quantum mechanically allowed 5-branes, p_r will be integers. The NS 5-brane corresponds to the charges $(1,0)$ and the D5-brane to $(0,1)$. The physical charges are obtained by multiplying with the 5-brane charge quantum μ. Apart from the charge, the NS 5-brane solution is characterised by the asymptotic value τ_∞. In the complex formalism, the solution for asymptotically vanishing dilaton and axion (i.e. $\tau_\infty = i$) is

$$\begin{aligned} ds^2 &= \Delta^{-1/4}dx^2 + \Delta^{3/4}dy^2\,, && \\ \mathcal{H}^0_{(3)} &= -\frac{i}{2}\,\Delta^{-1} *_y d\Delta\,, & H_{(5)} &= 0\,,\\ P &= \frac{i}{4}\,\Delta^{-1}d\Delta\,, & Q &= 0\,,\\ \mathcal{U}^1 &= i\,\Delta^{-1/4}\,, & \mathcal{U}^2 &= \Delta^{1/4}\,, \end{aligned} \tag{2.8}$$

where $\Delta = 1 + \mu/\rho^2$, ρ being the radial "distance" to the brane, and $*_{x,y}$ means dualisation with respect to the longitudinal (x) or the transverse (y) space. One advantage of the complex formalism is that the above expressions for the metric, the 3-form and the 1-forms contain all the (p,q) 5-branes. The point is that we have doublets $\mathcal{U}^r$ and $H_{(3)r}$ which transform under $\mathrm{SL}(2,\mathbb{R})$ as $\mathcal{U}^r \to M^r{}_s\mathcal{U}^s$, $H_{(3)r} \to (M^{-1})^s{}_r H_{(3)s}$, where M is an $\mathrm{SL}(2,\mathbb{R})$ matrix. When contracting the indices, we

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obtain an SL$(2,\mathbb{R})$-invariant object. Hence the 3-form $\mathcal{H}_{(3)}$ is the same for the NS 5-brane and the (p,q) 5-branes, as long as the background scalars are transformed along with the charges. If we want a specific type of brane, we can start with the NS 5-brane and then simultaneously make an SL$(2,\mathbb{Z})$ transformation on the scalar and 3-form doublets.

The general version of eq. (2.8) for arbitrary charge (p,q) and *arbitrary* background scalars is formally identical to (2.8) except for

$$\mathcal{U}^r = \epsilon^{rs}(-k\Delta^{1/4}p_s + ik^{-1}\Delta^{-1/4}\tilde{p}_s)\,, \tag{2.9}$$

where $\epsilon^{rs}p_r\tilde{p}_s = 1$ ($\tilde{p}_r$ does not have to be integer) and k is real, and where $\Delta = 1 + k^{-1}\mu/\rho^2$. Notice that $k^{-1} = |\mathcal{U}^r_\infty p_r|$, so the expression for Δ agrees with the one found in ref. [12]. The asymptotic value of the physical scalar is $\tau_\infty = -(kq - ik^{-1}\tilde{q})/(kp - ik^{-1}\tilde{p})$. Most of the explicit calculations will, for the sake of economy of notation, make use of the NS 5-brane charge $(p,q) = (1,0)$ and $\tau_\infty = i$, although the results will be stated in SL$(2,\mathbb{Z})$-covariant form.

3. Zero-mode solution

We start by analysing the zero-modes using the general Goldstone prescription introduced in ref. [1]. This analysis will tell us how the zero-modes are parametrised, in our case by a real 2- or 4-form in the longitudinal directions, and enable us to write down an ansatz for the exact solution. From the zero-mode solution we will also see exactly where the zero-modes sit in the target space fields.

As described in e.g. ref. [4], physical modes correspond to broken *large* gauge transformations. By a large gauge transformation we mean a transformation that does not go to zero at spatial infinity and therefore will change the charges associated with an object. If e.g. a large gauge transformation changes the momentum of an object we must conclude that the transformation corresponds to a time-dependent translation. In this sense, large gauge transformations are equivalent to global symmetries of the theory. By taking the equations of motion into account, as we will see later, we can determine the transversal dependence of the zero-modes and thus single out a particular large gauge transformation which corresponds to a global symmetry. We then have the ordinary correspondence between broken global symmetries and physical modes.

Small gauge transformations, i.e. transformations that go to zero at spatial infinity, on the other hand, relate equivalent configurations and therefore have nothing to do with global symmetries or zero-modes. They only represent a redundancy in our description of the theory.

It is important to note that since we are considering a theory with gravity, i.e. with reparametrisation invariance, in contrast to the ordinary monopole case, we must be careful when we talk about e.g. translations. To actually change the brane

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configuration we must do something that is more than a small gauge transformation and therefore have a non-vanishing effect at infinity. What we really mean by a "translation" in a theory with gravity is a *large* reparametrisation, and instead of saying that broken translational invariance gives rise to scalar modes on the brane we should say that the scalar modes arise by breaking the *large* reparametrisation invariance in the transverse directions [1]. Since we actually consider a theory with *super*gravity, the same arguing applies for the supersymmetry transformations [1].

We now introduce the tensor zero-modes as described in ref. [1], by making a gauge transformation $\delta C_{(2)r} = d\Lambda_{(1)r}$. Using the proposed mechanism to generate zero-modes as large gauge transformations, we make an ansatz $\mathcal{U}^r \Lambda_{(1)r} = \mathrm{Re}\,\mathcal{A}_{(1)}\Delta^{k_R} + i\,\mathrm{Im}\,\mathcal{A}_{(1)}\Delta^{k_I}$, where $\mathcal{A}_{(1)} = \mathcal{U}^r A_{(1)r}$ is a complex 1-form potential which lies in the longitudinal directions and $A_{(1)r}$ is *constant*. That the correct thing to do is to allow different radial behaviour for the real and imaginary parts is understood from the background values of $\mathcal{U}^r$. The reason why we take $\mathcal{A}_{(1)}$ to lie in the longitudinal directions is of course that we want to be able to integrate out the transversal dependence, thus obtaining an effective theory on the brane world-volume. We get

$$\delta\mathcal{C}_{(2)} = \mathrm{Re}\,\mathcal{A}_{(1)} \wedge d\Delta^{k_R} + i\,\mathrm{Im}\,\mathcal{A}_{(1)} \wedge d\Delta^{k_I}\,. \tag{3.1}$$

We now let $A_{(1)r}$ become x-dependent, which means that (3.1) is no longer a pure gauge transformation. By computing $\delta\mathcal{H}_{(3)}$ and solving the equations of motion for the variation we will get the equations of motion and the transversal behaviour of the zero-modes. We find (this relies on the observation that the scalars do not transform at linear level — this may be deduced group-theoretically or a posteriori from the solution)

$$\delta\mathcal{H}_{(3)} = \mathcal{U}^r d(\delta C_{(2)r}) = -\,\mathrm{Re}\,\mathcal{F}_{(2)} \wedge d\Delta^{k_R} - i\,\mathrm{Im}\,\mathcal{F}_{(2)} \wedge d\Delta^{k_I}\,, \tag{3.2}$$

where $\mathcal{F}_{(2)} = \mathcal{U}^r F_{(2)r} = \mathcal{U}^r dA_{(1)r}$ is the complex field strength on the brane. (Since type-IIB theory contains a doublet of 2-form potentials any $\mathrm{SL}(2,\mathbb{Z})$-covariant description of a brane must contain a doublet of vector potentials. How the complex field strength is related to the Born-Infeld field strength will be clarified below.) When solving the equations of motion it is convenient to have all the Δ-dependence explicit and we therefore introduce $\underline{\mathcal{F}}_{(2)}$, which is a Δ-independent complex 2-form when expressed in Lorentz indices, i.e. when we use the vielbeins to go from coordinate-frame to Lorentz indices. In the next section we need to introduce higher matrix powers of $\underline{\mathcal{F}}_{(2)}$. Using Lorentz indices, we then avoid the complexity of metric factors.

We then have

$$\delta\mathcal{H}_{(3)} = -\,\mathrm{Re}\,\underline{\mathcal{F}}_{(2)} \wedge d\Delta^{k_R} - i\,\mathrm{Im}\,\underline{\mathcal{F}}_{(2)} \wedge d\Delta^{k_I}\Delta^{-1/2}\,, \tag{3.3}$$

where we have taken the Δ factors coming from the vielbeins and from the $\mathcal{U}$'s into account.

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We also have to introduce zero-modes in $H_{(5)}$. This will be clear by looking at the result, but can be understood by the fact that not only strings but also 3-branes may end on 5-branes, the two types of couplings being related by what will manifest itself as a duality relation. Here, a slight complication is present. The Bianchi identity for the 5-form is readily solved by

$$H_{(5)} = dC_{(4)} + \frac{1}{2}\mathrm{Im}(\mathcal{C}_{(2)} \wedge \bar{\mathcal{H}}_{(3)})\,, \tag{3.4}$$

which is invariant under

$$\delta C_{(4)} = d\Lambda_{(3)} + \frac{1}{2}\mathrm{Im}(\Lambda_{(1)r}\mathcal{U}^r \wedge \bar{\mathcal{H}}_{(3)})\,. \tag{3.5}$$

The variation of $C_{(2)r}$ introduced in eq. (3.1) clearly affects $H_{(5)}$, in such a way, however, that the new 5-form field strength is not single-valued on the transverse 3-sphere (remember that the background value of $C_{(2)r}$ is such that $H_{(3)r}$ wrap the sphere with charges (p,q)). In order to use eq. (3.4) in a consistent way, i.e. in order to work with transformations at the level of gauge potentials, two things have to be achieved by the 3-form gauge parameter $\Lambda_{(3)}$. It must generate a "large" gauge transformation, *and* it must, when made x-dependent, cancel the otherwise inconsistent non-uniqueness in $H_{(5)}$. To be precise, the term we must include in $\Lambda_{(3)}$, in addition to the zero-modes, is

$$\frac{1}{2}\,\mathrm{Im}(\mathcal{C}_{(2)} \wedge \Lambda_{(1)r}\mathcal{U}^r)\,, \tag{3.6}$$

where $\mathcal{C}_{(2)}$ is the background value of the potential and the x-dependence of $\Lambda_{(1)r}$ is, as usual, turned on when the variation of $C_{(4)}$ is obtained. In the present case, where the background field strengths are purely transversal, the effect of this is that the "modification terms" in $H_{(5)}$ as well as in $\delta C_{(4)}$ are discarded, and one may formally proceed as if "$H_{(5)} = dC_{(4)}$". Would there have been longitudinal components of background field strengths, one had been led to consider modified field strengths also in brane directions, as in ref. [14].

We thus make a gauge variation $\delta C_{(4)} = d\Lambda_{(3)}$, where $H_{(5)} = dC_{(4)}$. Using the ansatz $\Lambda_{(3)} = \Delta^k A_{(3)}$, where $A_{(3)}$ is a constant 3-form potential in the longitudinal directions, yields

$$\delta C_{(4)} = -d\Delta^k \wedge A_{(3)}\,. \tag{3.7}$$

By letting $A_{(3)}$ become x-dependent we obtain

$$\delta H_{(5)} = -d\Delta^k \wedge G_{(4)} - *_y d\Delta^k \wedge *_x G_{(4)}\,, \tag{3.8}$$

where $G_{(4)} = dA_{(3)}$ and we have taken the requirement that $H_{(5)}$ must be self-dual into account. To make the Δ-dependence explicit we introduce $\underline{G}_{(4)}$, which is Δ-independent when expressed in Lorentz indices,

$$\delta H_{(5)} = -d\Delta^k\Delta^{-1/2} \wedge \underline{G}_{(4)} - \underline{*}_y d\Delta^k\Delta^{1/2} \wedge \underline{*}_x \underline{G}_{(4)}\,, \tag{3.9}$$

where $\underline{*}$ means dualisation with respect to the flat metric.

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By inserting the variations into the second Bianchi identity in (2.6) we get

$$d *_x \underline{G}_{(4)} = 0\,, \tag{3.10}$$

which is the equation of motion for $\underline{G}_{(4)}$. We also get a relation between $\underline{G}_{(4)}$ and $\underline{\mathcal{F}}_{(2)}$,

$$*_x\underline{G}_{(4)} = \frac{k_R}{2k(k_R - 1/4)}\,\mathrm{Re}\,\underline{\mathcal{F}}_{(2)}\,, \tag{3.11}$$

and the requirement that $k = k_R + 1/4$. The l.h.s. of the first identity in (2.6) vanishes to first order in $\underline{\mathcal{F}}_{(2)}$. Inserting the variations into the second equation in (2.5) yields the equation of motion for $\underline{\mathcal{F}}_{(2)}$,

$$d *_x \underline{\mathcal{F}}_{(2)} = 0\,, \tag{3.12}$$

and that $k_R = -3/4$ or $k_R = 5/4$ and $k_I = 0$ or $k_I = 3/4$. The l.h.s. of the first equation in (2.5) vanishes to first order in $\underline{\mathcal{F}}_{(2)}$.

Since a BPS brane in $D = 10$ has eight fermionic degrees of freedom, we must have eight bosonic degrees of freedom in order to have supersymmetry. The scalar fields contribute with four bosonic degrees of freedom for a 5-brane and we must get the remaining four from the tensor zero-modes. At first, it seems like we get too many from the tensor modes, since a complex 2-form field strength in six dimensions has eight degrees of freedom, i.e. twice the amount we wanted. We, however, require the zero-modes to be normalisable, which singles out $k_R = -3/4$ and $k_I = 0$, i.e. the imaginary part of $\delta\mathcal{H}_{(3)}$ vanishes. We have thus found that the zero-modes are parametrised by a real 2-form field strength, $\mathrm{Re}\,\underline{\mathcal{F}}_{(2)}$, which gives us the correct number of degrees of freedom.

We would like to comment in a more detailed way on the relation between the field strength "on the brane" and the background tensor potential. For an infinitesimally thin brane embedded in a target space, the equivalence is a consequence of the gauge invariance of "$F - B = dA - B$". In the context of a soliton solution, the brane is the entire field configuration between the horizon and Minkowski infinity, and the background fields are the values of the fields at infinity. Here we do not have F and B as independent objects, rather the behaviour of B is parametrised in terms of F. Take e.g. the linearised solution of the present section. For a constant $\mathrm{Re}\underline{\mathcal{F}}_{(2)}$ we may write the (interesting part of the) 3-form field strength as $\delta\mathcal{H}_{(3)} = d\,[-\mathrm{Re}\underline{\mathcal{F}}_{(2)}\Delta^{-3/4}]$. Note that the potential inside the square brackets goes to zero when ρ goes to zero (at the horizon) and to a finite background value when ρ goes to infinity. The configuration is that of an NS 5-brane in a constant (longitudinal) 2-form background. This was demonstrated here at linear level, but of course continues to hold to all orders for the solutions presented in the following section.

In a type-IIB theory, we are generically in a situation where there are several tensor potentials coupling to a brane. An important piece of information is the identification of the combination of tensor potentials that are related to the Born-Infeld

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degrees of freedom, and to the non-commutativity parameter. The parametrisation of the zero-modes, and later of the full solutions, in terms of a real 2-form provides exactly this information.

4. Finite field strength solution

The excitation of the 3-form is parametrised by the Δ-independent 2-form $F \equiv \mathrm{Re}\,\underline{\mathcal{F}}_{(2)}$, introduced in the previous section. The Goldstone analysis yields zero-modes which are linear in F, i.e. the analysis is only valid for small field strength. Finite constant field strength can be obtained by doing an expansion in F. However, we can truncate the expansion because of the Caley-Hamilton relation, which for antisymmetric matrices in 6-dimensional Minkowski space takes the form

$$F^6 = \frac{1}{2}\,t_2 F^4 + \left(\frac{1}{4}\,t_4 - \frac{1}{8}\,t_2^2\right) F^2 + \det(F)\,\eta\,, \tag{4.1}$$

where $t_2 = \mathrm{tr}(F^2)$, $t_4 = \mathrm{tr}(F^4)$ and η is the diagonal matrix corresponding to the Minkowski metric $(-,+,\ldots,+)$.

For the sake of simplicity, we start out with an NS 5-brane in $\tau_\infty = i$. The general case is kept under control as in section 2, and the $\mathrm{SL}(2,\mathbb{Z})$-covariant result will be stated at the end. The charges are determined by the real doublet of 3-forms $H_{(3)r}$. Once we excite the 3-form we must also excite the 1-form P and the 5-form to get solutions to the equations of motion. Since P is excited, the scalars will change. To stay on the $(1,0)$ brane, we need to change the complex background 3-form $\mathcal{H}^0_{(3)} = \mathcal{U}^r H^0_{(3)r}$ ("background" here meant in the sense "$F = 0$", not to be confused with asymptotic values).

We work in the gauge $\mathrm{Im}(\mathcal{U}^2) = 0$, corresponding to

$$\mathcal{U}^1 = e^{\phi/2}\chi + i\,e^{-\phi/2}\;, \quad \mathcal{U}^2 = e^{\phi/2}\,, \tag{4.2}$$

which forces us to have a non-vanishing $\mathrm{U}(1)$ gauge field (see the comment at the end of the section). Writing $ds^2 = g_{\mu\nu}dx^\mu dx^\nu + g_{pq}dy^p dy^q$, the longitudinal metric $g_{\mu\nu}$ will get F^2, F^4 and $t_2, t_4, \det(F)$ corrections, whereas the transverse metric $g_{pq} \equiv c^2\,\delta_{pq}$, just gets $t_2, t_4, \det(F)$ corrections, since F lies in the longitudinal directions. It is convenient to work with the matrix A, defined from the vielbein as $A \equiv \log e$ (where e means $e_\mu^{\;i}$). It is easy to get the metric from A. From the structure of the Einstein equations we get $c = \exp(-\,\mathrm{tr}(A))$, and the Ricci tensor now takes the simple form

$$\begin{aligned} R_{pq} &= -\partial_p\Delta\partial_q\Delta\left(\mathrm{tr}(A'^2) + \frac{1}{2}(\mathrm{tr}\,A')^2\right) + \frac{1}{2}\delta_{pq}(\partial\Delta)^2\,\mathrm{tr}\,A''\,,\\ R_{\mu\nu} &= -c^{-2}(\partial\Delta)^2 e_\mu{}^i e_\nu{}^j A''_{ij}\,, \end{aligned} \tag{4.3}$$

where prime indicates differentiation with respect to Δ.

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The coordinate dependence of the solution can be written in terms of Δ, and the ansatz thus takes the form

$$\begin{aligned} A &= a_0\, \eta + a_2\, F^2 + a_4\, F^4\,, \\ \mathcal{H}_{(3)} &= h *_y d\Delta - \mathcal{F}_2 \wedge d\Delta\,, \\ H_{(5)} &= G_{(2)} \wedge *_y d\Delta - d\Delta \wedge *_x G_{(2)}\,, \\ P &= p\, d\Delta\,, \qquad Q = q\, d\Delta\,, \end{aligned} \tag{4.4}$$

where

$$\begin{aligned} (\mathcal{F}_{(2)})_{ij} &= f_1\, F_{ij} + f_3\, F^3_{ij} + f_5\, F^5_{ij}\,, \\ (G_{(2)})_{ij} &= g_1\, F_{ij} + g_3\, F^3_{ij} + g_5\, F^5_{ij} \end{aligned} \tag{4.5}$$

and we have chosen to write the 5-form in terms of the 2-form

$$G_{(2)} = - * G_{(4)}\,. \tag{4.6}$$

It is a straight-forward group-theoretical excercise to convince oneself that no other independent combinations of F's can enter the ansatz. Q is pure gauge, since the form of P yields $dQ = 0$. The Maurer-Cartan equations are therefore automatically satisfied. Here p, q, h, f_1, f_3 and f_5 are complex functions of Δ, t_2, t_4 and $\det(F)$, and a_0, a_2, a_4, g_1, g_3 and g_5 are real functions of the same variables and parameters. The functions p and q should of course not be confused with the $SL(2,\mathbb{Z})$ charges (p, q).

Inserting the ansatz in the equations of motion and Bianchi identities yields equations for these functions, see appendix A. The background solution corresponds to

$$a_0 = -\frac{1}{8}\log\Delta\,, \qquad p = \frac{i}{4}\Delta^{-1}\,, \qquad h = -\frac{i}{2}\Delta^{-1}\,, \tag{4.7}$$

and the rest zero. The first-order correction is the zero-mode solution found in the previous section, $f_1, g_1 \sim \Delta^{-3/2}$, where the Δ-dependence of the vielbeins has been taken into account. Now we can solve the equations order by order and the result is expansions in negative powers of Δ, see appendix A. Because of the complexity of the equations and the ansatz, we made a Mathematica program to get the expansion of the solution. We were able to sum up these series and we have checked the result explicitly for the P equation of motion. The solution can be written in terms of the following functions:

$$\begin{aligned} f_{2+} &\equiv \Delta + \frac{9}{16}\, t_2\,, \\ f_{2-} &\equiv \Delta - \frac{9}{16}\, t_2\,, \\ f_4 &\equiv \Delta^2 + \left(\frac{9}{16}\right)^2 ((t_2)^2 - 4t_4)\,, \\ f_{\det} &\equiv f_{2+} f_4 - \left(\frac{27}{8}\right)^2 \det(F)\,. \end{aligned} \tag{4.8}$$

Only the first four terms in the expansions were used to determine the solution. For half of the functions in the ansatz we have calculated 8-9 more terms and for the

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other half 21-22 more terms, and they all agree with the solution written in closed form. We therefore feel confident that the solution is indeed the exact one.

We converted the expansions for a_0, a_2 and a_4 to expansions for the metric with the following exact result

$$
\begin{aligned}
g_{\mu\nu} &= (f_{\rm det})^{-3/4} f_4\, \eta_{\mu\nu} + \frac{9}{4}\,(f_{\rm det})^{-3/4} f_{2-} (F^2)_{\mu\nu} + \left(\frac{9}{4}\right)^2 (f_{\rm det})^{-3/4} (F^4)_{\mu\nu}\,, \\
c^2 &= (f_{\rm det})^{1/4}\,,
\end{aligned} \tag{4.9}
$$

where $F_{\mu\nu} = \delta^i_\mu \delta^j_\nu F_{ij}$. For the 5-form, we get the remarkable result that g_3 and g_5 are identically zero, and therefore $G_{(2)}$ is proportional to F

$$
(G_{(2)})_{ij} = \frac{3}{8}\,(f_{\rm det})^{-1/2} F_{ij}\,. \tag{4.10}
$$

The 3-form is specified by

$$
\begin{aligned}
h &= -\frac{1}{2}\,(f_4)^{-1/2} \left(\frac{27}{8}\,\mathrm{pf}(F)\,(f_{\rm det})^{-1/2} + i\right), \\
f_1 &= -\frac{3}{4}\,(f_{\rm det})^{-1/2} (f_4)^{-1/2} f_{2-} + i\,\frac{2}{9\,\mathrm{pf(F)}} \left((f_{\rm det})^{-1} (f_4)^{3/2} - (f_4)^{-1/2} f_{2-}\right), \\
f_3 &= -\frac{27}{16}\,(f_{\rm det})^{-1/2} (f_4)^{-1/2} - i\,\frac{1}{2\,\mathrm{pf(F)}}\,(f_4)^{-1/2} \left(1 - (f_{\rm det})^{-1} f_4 f_{2-}\right), \\
f_5 &= i\,\frac{9}{8\,\mathrm{pf(F)}}\,(f_{\rm det})^{-1} (f_4)^{1/2}\,,
\end{aligned} \tag{4.11}
$$

where we have introduced the pfaffian, which satisfies $\mathrm{pf}(F)^2 = \det(F)$. The 1-forms are

$$
\begin{aligned}
q &= \frac{27}{8}\,\Delta\,\mathrm{pf}(F)\,(f_{\rm det})^{-1/2} (f_4)^{-1}\,, \\
p &= i\,\Delta \left((f_4)^{-1} - \frac{1}{4}(2\Delta f_{2+} + f_4)(f_{\rm det})^{-1}\right) + \frac{27}{8}\,\Delta\,\mathrm{pf}(F)\,(f_{\rm det})^{-1/2} (f_4)^{-1}\,.
\end{aligned} \tag{4.12}
$$

The above solution for the metric, the 5-form, the 3-form and the 1-forms constitutes the $\mathrm{SL}(2,\mathbb{Z})$-covariant part of the solution and is therefore valid for all (p,q) 5-branes. The only remaining information lies in the background value of the scalars. For instance, the NS 5-brane with $\tau_\infty = i$ has

$$
\begin{aligned}
\chi &= \frac{27}{8}\,\mathrm{pf}(F)\,(f_4)^{-1}\,, \\
e^{-\phi} &= (f_{\rm det})^{1/2} (f_4)^{-1}\,,
\end{aligned} \tag{4.13}
$$

which in our choice of gauge corresponds to the scalar doublet

$$
\begin{aligned}
\mathcal{U}^1 &= (f_{\rm det})^{1/4} (f_4)^{-1/2} \left(\frac{27}{8}\,\mathrm{pf}(F)\,(f_{\rm det})^{-1/2} + i\right), \\
\mathcal{U}^2 &= (f_{\rm det})^{-1/4} (f_4)^{1/2}\,.
\end{aligned} \tag{4.14}
$$

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It is straightforward to get the doublet of 3-forms from $\mathcal{H}_{(3)}$ and the scalar doublet. As explained in section 2, a (p,q) 5-brane can be obtained by simultaneously making $\mathrm{SL}(2,\mathbb{Z})$ transformations on these two doublets. Then we of course just get the 5-branes obtained as the $\mathrm{SL}(2,\mathbb{Z})$ orbit on $\tau_\infty = i$. For the solution with general charges and arbitrary background scalars, the scalar doublet is the finite field strength modification of (2.9), leading to the covariant expression

$$\mathcal{U}^r = \epsilon^{rs}\left(k^{-1}\tilde{p}_s(f_{\mathrm{det}})^{1/4}(f_4)^{-1/2}\left(\frac{27}{8}\,\mathrm{pf}(F)(f_{\mathrm{det}})^{-1/2}+i\right) - kp_s(f_{\mathrm{det}})^{-1/4}(f_4)^{1/2}\right). \tag{4.15}$$

With the covariant gauge choice discussed below, we thus have an $\mathrm{SL}(2,\mathbb{Z})$-covariant solution.

To linear order the exact solution reduces to the zero-mode solution found in section 3. For all the fields, except the 3-form on the brane, it can easily be checked that we obtain the background NS 5-brane solution by setting F and the parameters depending on F to zero. For the 3-form on the brane we have to be more careful, since we divide by the pfaffian in the imaginary parts. If we choose a certain basis for F, see the next section, and insert f_1, f_3 and f_5 into $\mathcal{F}_{(2)}$, it can be seen that $\mathcal{F}_{(2)}$ will indeed vanish when F does.

A comment on U(1) gauge choices: The solutions for vanishing field strength, given in eqs. (2.8) and (2.9), use $Q=0$ as a choice of gauge for the local U(1) symmetry. This gauge choice has the advantage of being manifestly $\mathrm{SL}(2,\mathbb{Z})$-invariant. In the case of an NS5-brane, it is equivalent to $\mathrm{Im}(\mathcal{U}^2)=0$. The reason for our departure from $Q=0$ when the tensor is turned on is purely practical. The field configuration is always such that P points in the radial direction, which means that Q is closed, $dQ=0$, and it is formally trivial to perform a gauge transformation to reach $Q=0$. The difficulty we encounter for finite field strength is that even though we have an explicit expression for Q, eq. (4.12), we have not been able to integrate it to a closed analytic expression. The gauge choice $\mathrm{Im}(\mathcal{U}^2)=0$ is therefore better suited to our ambition of giving closed explicit solutions for all fields. It is not $\mathrm{SL}(2,\mathbb{Z})$-covariant as it stands, but can easily be made so with the introduction of general charges — it then reads $\mathrm{Im}(\mathcal{U}^r\tilde{p}_r)=0$. The general solution (4.15) clearly fulfills this equation.

5. Properties of the metric

In this section we analyse the metric arising from the deformation of the 5-brane via the finite constant field strength. To get a solution which makes sense, we must impose a condition on the parameters of this field strength.

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Since F is antisymmetric, we can find a basis where it takes the simple form

$$F_{ij} = \begin{pmatrix} 0 & \nu_1 & 0 & 0 & 0 & 0 \\ -\nu_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \nu_2 & 0 & 0 \\ 0 & 0 & -\nu_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \nu_3 \\ 0 & 0 & 0 & 0 & -\nu_3 & 0 \end{pmatrix} . \tag{5.1}$$

This is defined in Minkowski space and therefore we get a sign change in t_2 compared to the trace in euclidean space

$$\begin{aligned} t_2 &= 2\nu_1^2 - 2\nu_2^2 - 2\nu_3^2 \,, & t_4 &= 2\nu_1^4 + 2\nu_2^4 + 2\nu_3^4 \,, \\ \mathrm{pf}(F) &= -\nu_1\,\nu_2\,\nu_3 \,, & \det(F) &= \nu_1^2\,\nu_2^2\,\nu_3^2 \,. \end{aligned} \tag{5.2}$$

In this basis, the metric in the brane directions will be diagonal, since F^2 and F^4 are diagonal. Furthermore, from the structure of F, we get a natural split into three two-dimensional spaces, one minkowskian and two euclidean. The components of the metric in the subspaces are

$$\begin{aligned} g_{11} &= (f_{det})^{-3/4} \left(\Delta \left(\Delta + \frac{9}{4}\nu_1^2 \right) + \left(\frac{9}{8} \right)^2 (\nu_1^4 - (\nu_2^2 - \nu_3^2)^2) \right) , \\ g_{33} &= (f_{\mathrm{det}})^{-3/4} \left(\Delta \left(\Delta - \frac{9}{4}\nu_2^2 \right) + \left(\frac{9}{8} \right)^2 (\nu_2^4 - (\nu_1^2 + \nu_3^2)^2) \right) , \\ g_{55} &= (f_{\mathrm{det}})^{-3/4} \left(\Delta \left(\Delta - \frac{9}{4}\nu_3^2 \right) + \left(\frac{9}{8} \right)^2 (\nu_3^4 - (\nu_1^2 + \nu_2^2)^2) \right) , \end{aligned} \tag{5.3}$$

where f_{det}, expressed in the ν parameters, becomes

$$\begin{aligned} f_{\mathrm{det}} = &\left(\Delta + \frac{9}{8}(\nu_1^2 - \nu_2^2 - \nu_3^2) \right) \left(\Delta^2 - \frac{81}{64} \left(\nu_1^4 + (\nu_2^2 - \nu_3^2)^2 + 2\nu_1^2(\nu_2^2 + \nu_3^2) \right) \right) - \\ &- \left(\frac{27}{8} \right)^2 \nu_1^2\,\nu_2^2\,\nu_3^2 \,. \end{aligned} \tag{5.4}$$

Besides from the advantage of having a diagonal metric, one also has detailed knowledge about F. When all the ν's are non-vanishing, the matrix has full rank, and lower rank can be obtained by turning one or two of the parameters off. The exact solution found in the previous section is of course simplified when using a lower rank matrix. If we for instance set ν_2 and ν_3 to zero, everything can be expressed in terms of just two functions $\Delta \pm \frac{9}{8}\nu_1$.

In the limit of vanishing 2-form field strength, the longitudinal metric has the usual asymptotic Minkowski form $(-,+,+,+,+,+)$. For finite field strength, the metric should still asymptotically be Minkowski, i.e. the first component should be

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negative and the rest should be positive. This gives a restriction on the ν parameters. Because of the negative power of $f_{\rm det}$ in the metric, in order to get a non-singular solution, the parameters should fulfill yet another restriction. This is related to the breakdown of Born-Infeld dynamics at some finite field strength. It turns out that the latter requirement is the stronger, and we get

$$\nu_1^2 + \nu_2^2 + \nu_3^2 \leq \frac{8}{9} \, . \tag{5.5}$$

In ref. [2], we calculated the finite tensor deformations of the D3 and the M5-branes. Because of the duality properties, the solutions just depend on one parameter ν, which also has an upper bound. In the limiting case, the metric blows up asymptotically in some directions on the brane and shrinks in the other directions. This was interpreted as smeared out strings or membranes. Here we get the same kind of behaviour in the limiting case $\nu_1^2 + \nu_2^2 + \nu_3^2 = 8/9$. For almost every possibility (the exception is considered below), after trivial rescalings, the leading asymptotic terms in the metric become

$$ds^2 = \rho^{3/2} dx_1^2 + \rho^{-1/2}(dx_2^2 + dx_3^2 + dy^2) \, , \tag{5.6}$$

where dx_i^2, $i = 1, 2, 3$, corresponds to the three two-dimensional subspaces on the brane. We see that the metric in the two-dimensional minkowskian subspace blows up asymptotically, whereas it shrinks in the other directions, and the result is a smeared out string. The exception is when just ν_2 or ν_3 is different from zero. Considering the latter, we get the following asymptotical behaviour of the metric

$$ds^2 = \rho \, (dx_1^2 + dx_2^2) + \rho^{-1}(dx_3^2 + dy^2) \, . \tag{5.7}$$

Here four directions blow up and the rest shrink. This is interpreted as a smeared out 3-brane.

In summary, we saw in the previous section that F and its higher powers were used as a basis, and the solution was described in terms of the traces and the determinant. Using the frame introduced in this section, various properties of the solution become more clear. The metric becomes diagonal and we have seen that we get an upper bound on the parameters of the finite constant field strength.

6. Discussion

We have constructed explicitly the type-IIB supergravity solutions corresponding to 5-branes with arbitrary charges and with finite Born-Infeld field strength, or equivalently, in a finite background tensor field. The solutions are considerably more involved than in the previously treated cases of type-IIB 3-branes and M5-branes [2], due to the fact that a (p, q) 5-brane couples both to the NS-NS and RR 2-forms and to the chiral 4-form, which in the solutions manifests itself as excitations of all these tensors.

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One of the main results of the paper is that it answers the question provoked by the title — what is "the B-field" that a (p, q) 5-brane feels? We have parametrised the physical modes by a real 2-form, which identifies the unique combination of the doublet of 2-forms and the 4-form that can not be gauged away in the presence of a (p, q) 5-brane.

We envisage some potential applications of the result. The first one, which was one of the main reasons for initiating this work, concerns the $\mathrm{SL}(2, \mathbb{Z})$-covariant formulation of type-IIB 5-brane dynamics, continuing the work of refs. [13, 14] for lower-dimensional type-IIB branes. Related issues have been addressed in a number of papers [3, 14, 12, 15]. A straight-forward Poincaré dualisation of the Born-Infeld action was first attempted in ref. [3]. It is however unclear how relevant such a procedure is — the Born-Infeld field strength, being related to the NS-NS 2-form, is not $\mathrm{SL}(2, \mathbb{Z})$-invariant and the dual can hardly be associated with the $\mathrm{SL}(2, \mathbb{Z})$-invariant 4-form. In ref. [14], it was clarified how a duality relation between a complex 2-form and a real 4-form will be a natural ingredient in the type of description we are looking for. For all values of the radius, there will be a (radius-dependent) duality relation. It is likely that this relation may be deduced using the results of the present paper. The only obstacle could be that one encounters equations that do not have obvious analytic solutions, as in ref. [3]. We find it more likely, however, that this will happen when one tries to solve for some tensors in terms of others. The attempt of ref. [3] to dualise the Born-Infeld field failed due to the occurrence of quintic equations. Here, we also note that both the 2-form and 4-form field strengths on the brane are explicitly parametrised by a real 2-form (F), which, in light of the surprisingly simple expression (4.10) for the 4-form (for which we have no other explanation than that we were lucky in our choice of basis), means that the complex 2-form $\mathcal{F}$ is easily expressed in terms of the 4-form G. The opposite, to express G in terms of $\mathcal{F}$ would demand solving a quintic equation. An order-by-order solution of the problem of finding an $\mathrm{SL}(2, \mathbb{Z})$-covariant action was attempted in ref. [15], with partial success.

It is well known that the Born-Infeld action (and similar actions, e.g. for chiral 2-forms in $D = 6$) has a maximal finite field strength where the generalisation of the volume element containing $\det(g + F)$ degenerates, and the canonical electric field diverges. Analogously, there is a maximal value of the non-commutativity parameter in non-commutative Maxwell theory. The solutions presented here and in ref. [2] go further — they investigate in which sense 10- or 11-dimensional space-time degenerates in this limit. The result is, maybe somewhat surprisingly, that the limiting space-times are well-defined, but with an asymptotic structure that differs radically from the asymptotically Minkowski brane solutions. There is an effective reduction of the dimensionality of the boundary (boundary understood in the same sense as in AdS space), corresponding to a lower-dimensional brane. In the present paper string and 3-brane degeneracies were presented explicitly, which gives an even stronger indication that the limiting cases in some not yet fully understood sense correspond to "branes-within-branes" situations. It is conceivable that there is an AdS/CFT-like correspondence between superstring theory on these spaces and field theories on the (codimension > 1) boundaries.

Finally, the solutions found here and in ref. [2] should be relevant in connection to non-commutative geometry [16]. It has already been stressed that a finite value of the brane field strength is identical to a situation with finite background B field (due to the gauge transformation $\delta B = d\Lambda$, $\delta A = \Lambda$, leaving the combination $dA - B$ invariant). The results of this paper contain explicit information, presented in an $\mathrm{SL}(2,\mathbb{Z})$-covariant form, about where among the various tensor fields in type-IIB supergravity the non-commutativity parameter is to be found. The corresponding information for the 3-brane is found in ref. [2]. Although the full solution for the M5-brane is known, we do not know the generalisation of non-commutativity relevant for (chiral) 2-forms, and in that sense the M5-brane is still concealing some, probably very interesting and stringy, secrets. To our knowledge, non-commutative field theory has never been treated with the ambition of manifesting $\mathrm{SL}(2,\mathbb{Z})$ — this may prove interesting for e.g. the 3-brane and 5-branes in type-IIB, and the present techniques would undoubtedly be relevant. Neither has any concrete understanding been obtained concerning generalisations to higher rank tensors. Maybe detailed knowledge of the target space configurations of the type presented here and in ref. [2] can be helpful.

A. Equations and expansions

In this appendix we write down the equations obtained by insertion of the finite tensor ansatz and the resulting solution, obtained using Mathematica, as expansions in negative powers of Δ. The equations are

$$\begin{aligned}
p' - 2i\,qp - i\,h^2 + \frac{i}{2}\,\mathrm{tr}(\mathcal{F}_{(2)}^2) &= 0\,,\\
\mathcal{F}'_{(2)} - 2\,A'\mathcal{F}_{(2)} + i\,q\mathcal{F}_2 - i\,p\bar{\mathcal{F}}_{(2)} - 4i\,hG_{(2)} - 4i\,G_{(2)} \star \mathcal{F}_{(2)} &= 0\,,\\
h' - h\,\mathrm{tr}(A') - i\,qh + i\bar{h}p &= 0\,,\\
2\,G_{(2)}A' + G'_{(2)} - \mathrm{tr}(A')G_{(2)} + \mathrm{Im}(h\mathcal{F}_{(2)}) &= 0\,,\\
\mathrm{tr}\,((A')^2) + \frac{1}{2}\,(\,\mathrm{tr}(A'))^2 + 2\,|p|^2 - \mathrm{tr}(\bar{\mathcal{F}}_{(2)}\mathcal{F}_{(2)}) + 4\,\mathrm{tr}(G_{(2)}^2) - 2\,|h|^2 &= 0\,,\\
A'' - 2\,\bar{\mathcal{F}}_{(2)}\mathcal{F}_{(2)} + \frac{1}{4}\,\mathrm{tr}(\bar{\mathcal{F}}_{(2)}\mathcal{F}_{(2)})\,\eta - \frac{1}{2}\,|h|^2\,\eta + 2\,\mathrm{tr}(G_{(2)}^2)\,\eta - 8\,G_{(2)}^2 &= 0\,,
\end{aligned} \tag{A.1}$$

where $(G_{(2)} \star \mathcal{F}_{(2)})_{ij} = \frac{1}{4}\,\epsilon_{ijklmn}G_{(2)}{}^{kl}\mathcal{F}_{(2)}{}^{mn}$, and prime means differentiation with respect to Δ. The first two equations are the equations of motion for P and $\mathcal{H}_{(3)}$,

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respectively. The next two are the Bianchi identities for $\mathcal{H}_{(3)}$ and $H_{(5)}$, and the last two are the Einstein equations. In fact, there is a redundancy in these equations, since e.g. the first Einstein equation differentiated yields a combination of the others.

The above equations are written on a compact form, since we really should split them according to the basis elements F, F^3 and F^5 for the antisymmetric matrices and η, F^2 and F^4 for the symmetric matrices. The first few terms in the solution are

$$\begin{aligned}
a_0 &= -\frac{1}{8}\log\Delta - \frac{3}{8}\frac{9}{16}\,t_2\,\Delta^{-1} + \frac{1}{16}\left(\frac{9}{16}\right)^2 (5t_2^2 - 8t_4)\,\Delta^{-2} + \cdots\,,\\
a_2 &= \frac{9}{8}\,\Delta^{-1} - \frac{9}{8}\frac{9}{16}\,t_2\,\Delta^{-2} + \frac{3}{8}\left(\frac{9}{16}\right)^2 (t_2^2 + 4t_4)\,\Delta^{-3} + \cdots\,,\\
a_4 &= \left(\frac{9}{8}\right)^2 \Delta^{-2} - \frac{27}{32}\frac{9}{16}\,t_2\,\Delta^{-3} + \frac{9}{32}\left(\frac{9}{16}\right)^2 t_4\,\Delta^{-4} + \cdots\,,\\
f_1 &= -\frac{3}{4}\,\Delta^{-3/2}\left(1 + \frac{3}{2}\frac{9}{16}\,t_2\,\Delta^{-1} + \cdots\right) + i\,\frac{1}{16}\frac{9}{16\,\mathrm{pf}(F)}\,(3t_2^2 - 4t_4)\,\Delta^{-2} + \cdots\,,\\
f_3 &= i\,\frac{1}{\mathrm{pf}(F)}\Delta^{-2}\left(-\frac{9}{16}\,t_2 + \left(\frac{9}{16}\right)^2 t_2^2\,\Delta^{-1} + \cdots\right) - \frac{27}{16}\,\Delta^{-5/2} + \cdots\,,\\
f_5 &= i\,\frac{9}{8\,\mathrm{pf}(F)}\Delta^{-2}\left(1 - \frac{9}{16}\,t_2\,\Delta^{-1} + \frac{1}{2}\left(\frac{9}{16}\right)^2 (t_2^2 + 4t_4)\,\Delta^{-2} + \cdots\right),\\
g_1 &= \frac{3}{8}\,\Delta^{-3/2}\left(1 - \frac{9}{16}\,t_2\,\Delta^{-1} - \frac{3}{8}\left(\frac{9}{16}\right)^2 (t_2^2 - 16t_4)\,\Delta^{-2} + \cdots\right),\\
g_3 &= g_5 = 0 + \cdots\,,\\
p &= i\,\frac{1}{4}\,\Delta^{-1}\left(1 + \frac{9}{16}\,t_2\,\Delta^{-1} + \cdots\right) + \frac{27}{8}\,\mathrm{pf}(F)\,\Delta^{-5/2} + \cdots\,,\\
q &= \frac{27}{8}\,\mathrm{pf}(F)\,\Delta^{-5/2}\left(1 - \frac{1}{2}\frac{9}{16}\,t_2\,\mathrm{pf}(F)\,\Delta^{-1} + \cdots\right).
\end{aligned} \tag{A.2}$$

From the expansion of p and q we get an expansion of the scalar doublet $\mathcal{U}^r$, using the constraint and the gauge choice. The result is, for charges $(1,0)$ and $\tau_\infty = i$,

$$\begin{aligned}
\mathcal{U}^1 &= i\,\Delta^{-1/4} + i\,\frac{1}{4}\frac{9}{16}\,t_2\,\Delta^{-5/4} + \frac{27}{8}\,\mathrm{pf}(F)\,\Delta^{-7/4} + \cdots\,,\\
\mathcal{U}^2 &= \Delta^{1/4} - \frac{1}{4}\frac{9}{16}\,t_2\,\Delta^{-3/4} + \frac{1}{32}\left(\frac{9}{16}\right)^2 (13t_2^2 - 32t_4)\,\Delta^{-7/4} + \cdots\,.
\end{aligned} \tag{A.3}$$

By calculating the charges, it can be checked that we still have a $(1,0)$-brane with $\tau_\infty = i$. Covariantisation is straight-forward.

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References

[1] T. Adawi, M. Cederwall, U. Gran, B.E. W. Nilsson and B. Razaznejad, *Goldstone tensor modes*, *J. High Energy Phys.* **02** (1999) 001 [hep-th/9811145].

[2] M. Cederwall, U. Gran, M. Holm and B.E. W. Nilsson, *Finite tensor deformations of supergravity solitons*, *J. High Energy Phys.* **02** (1999) 003 [hep-th/9812144].

[3] M. Aganagic, J. Park, C. Popescu and J.H. Schwarz, *Dual D-brane actions*, *Nucl. Phys.* **B 496** (1997) 215 [hep-th/9702133].

[4] R. Rajaraman, *Solitons and instantons*, North-Holland, 1982;
J.A. Harvey, *Magnetic monopoles, duality and supersymmetry*, hep-th/9603086.

[5] M. Cederwall, A. von Gussich, B.E. W. Nilsson and A. Westerberg, *The Dirichlet super-three-brane in ten-dimensional type-IIB supergravity*, *Nucl. Phys.* **B 490** (1997) 163 [hep-th/9610148];
M. Cederwall, A. von Gussich, B.E. W. Nilsson, P. Sundell and A. Westerberg, *The dirichlet super-p-branes in ten-dimensional type-IIA and -IIB supergravity*, *Nucl. Phys.* **B 490** (1997) 179 [hep-th/9611159];
M. Aganagic, C. Popescu and J.H. Schwarz, *D-brane actions with local kappa symmetry*, *Phys. Lett.* **B 393** (1997) 311 [hep-th/9610249]; *Gauge-invariant and gauge-fixed D-brane actions*, *Nucl. Phys.* **B 495** (1997) 99 [hep-th/9612080];
E. Bergshoeff and P.K. Townsend, *Super D-branes*, *Nucl. Phys.* **B 490** (1997) 145 [hep-th/9611173].

[6] C.G. Callan, Jr., J.A. Harvey and A. Strominger, *Worldbrane actions for string solitons*, *Nucl. Phys.* **B 367** (1991) 60;
D.M. Kaplan and J. Michelson, *Zero modes for the $D = 11$ membrane and five-brane*, *Phys. Rev.* **D 53** (1996) 3474 [hep-th/9510053].

[7] E. Witten, *String theory dynamics in various dimensions*, *Nucl. Phys.* **B 443** (1995) 85 [hep-th/9503124];
C.M. Hull and P.K. Townsend, *Unity of superstring dualities*, *Nucl. Phys.* **B 438** (1995) 109 [hep-th/9410167];
J.H. Schwarz, *The power of M-theory*, *Phys. Lett.* **B 367** (1996) 97 [hep-th/9510086];
A. Sen, *Unification of string dualities*, *Nucl. Phys.* **58** *(Proc. Suppl.)* (1997) 5 [hep-th/9609176];
P.K. Townsend, *Four lectures on M-theory*, hep-th/9612121.

[8] M.J. Duff, R.R. Khuri and J.X. Lu, *String solitons*, *Phys. Rep.* **259** (1995) 213 [hep-th/9412184].

[9] J.H. Schwarz, *Covariant field equations of chiral $N = 2$ $D = 10$ supergravity*, *Nucl. Phys.* **B 226** (1983) 269;
P.S. Howe and P.C. West, *The complete $N = 2$, $D = 10$ supergravity*, *Nucl. Phys.* **B 238** (1984) 181.

JHEP01(2000)037

[10] J.H. Schwarz, *An* $\mathrm{SL}(2,\mathbb{Z})$ *multiplet of type-IIB superstrings*, *Phys. Lett.* **B 360** (1995) 13 [hep-th/9508143].

[11] M.J. Duff, *Supermembranes*, hep-th/9611203.

[12] J.X. Lu and S. Roy, *On the construction of* $\mathrm{SL}(2,\mathbb{Z})$ *type-IIB 5-branes*, hep-th/9812011.

[13] M. Cederwall and P.K. Townsend, *The manifestly* $\mathrm{SL}(2,\mathbb{Z})$*-covariant superstring*, *J. High Energy Phys.* **09** (1997) 003 [hep-th/9709002].

[14] M. Cederwall and A. Westerberg, *World-volume fields,* $\mathrm{SL}(2,\mathbb{Z})$ *and duality: the type-IIB 3- brane*, *J. High Energy Phys.* **02** (1998) 004 [hep-th/9710007].

[15] A. Westerberg and N. Wyllard, *Towards a manifestly* $\mathrm{SL}(2,\mathbb{Z})$*-covariant action for the type-IIB* (p,q) *super-five-branes*, *J. High Energy Phys.* **06** (1999) 006 [hep-th/9905019].

[16] N. Seiberg and E. Witten, *String theory and noncommutative geometry*, *J. High Energy Phys.* **09** (1999) 032 [hep-th/9908142].

JHEP01(2000)037